Received: *May 9, 2001*, Accepted: *July 2, 2001*
Revised : *June 29, 2001*

Lepton-flavor violation with non-universal soft terms

Daniel F. Carvalho and Mario E. Gómez

CFIF, Departamento de Física, Instituto Superior Técnico
Av. Rovisco Pais
1049-001 Lisboa, Portugal
E-mail: dani@gtae3.ist.utl.pt, mgomez@cfif.ist.utl.pt

Shaaban Khalil

Centre for Theoretical Physics, University of Sussex
Brighton BN1 9QJ, U.K. and
Ain Shams University, Faculty of Science
Cairo, 11566, Egypt
E-mail: s.khalil@sussex.ac.uk

Abstract: We study the lepton-flavor violation processes $\tau \to \mu\gamma$ and $\mu \to e\gamma$ in two different examples of models with non-universal soft breaking terms derived from strings. We show that the predictions are quite different from those of universal scenarios. Non-universal A-terms provide an interesting framework to enhance the supersymmetric contributions to CP violation effects. We observe that in the case of the lepton-flavor violation we study, the non-universality of the scalar masses enhances the branching ratios more significantly than the non-universality of the A-terms. We find that the current experimental bounds on these processes restrict both the parameter space of the models and the texture of the Yukawa couplings which predicts the lepton masses, providing at the same time an interesting experimental test for physics beyond the Standard Model.

Keywords: Supersymmetric Standard Model, Rare Decays.

Contents

1. Introduction

Flavor changing neutral currents (FCNC's) provide an important test for any new physics beyond the Standard Model (SM). It is well known that in the SM both baryon and lepton numbers are automatically conserved and that tree-level flavor-changing neutral currents are absent. After the recent result from Super-Kamiokande, which is regarded as compelling evidence for the oscillation of ν_μ to ν_τ with a squared mass difference of order $10^{-3} - 10^{-2}$ eV^2 [1, 2], there has been crescent interest in lepton-flavor violation (LFV). The rates for charged LFV processes are extremely small in the SM with right-handed neutrinos ($\propto \Delta m_\nu^2/M_W^2$ [3]). The present experimental limits [4] are

$$\begin{aligned} BR(\mu \to e\gamma) &< 1.2 \times 10^{-11} \\ BR(\tau \to \mu\gamma) &< 1.1 \times 10^{-6} \\ BR(\tau \to e\gamma) &< 2.7 \times 10^{-6}\,. \end{aligned} \tag{1.1}$$

Thus, the observation of these processes would be a signal of new physics. In different extensions of the SM, predictions of LFV processes compatible with the above experimental limits have been studied. For instance, supersymmetric (SUSY) models with a gauge unification group (GUT) and SUSY models with "see-saw" neutrinos have been discussed in refs. [5]–[8], SUSY models with R-parity violation in ref. [9], and recently, models with extra dimensions in ref. [10]. LFV can be observed in other systems, such as $\mu \to e$ conversion on heavy nuclei and $\mu \to 3e$ [6, 7], rare kaon decays [11], and slepton flavor mixing at future accelerator experiments [12]. However, the first two processes listed above are the most restrictive in the models considered in this work; the predictions for $BR(\tau \to e\gamma)$ are typically much lower than the bound (1.1).

In R-parity conserving SUSY models, the presence of LFV processes is associated with vertices involving leptons and their superpartners [13]. In the Minimal Supersymmetric Standard Model (MSSM), with universal soft terms, it is possible to rotate the charged lepton Yukawa couplings and the sleptons in such a way that lepton flavor is preserved. However, a small deviation from flavor universality in the soft-terms at the GUT scale will be severely constrained by the experimental bounds (1.1). In fact GUT theories [5] and models with U(1) family symmetries [8] can lead to the MSSM with flavor-dependent soft terms leading to important violations of the lepton flavor.

String-inspired models naturally lead to such non-universality in the soft SUSY breaking sector [14, 15, 16], and in such models a non-diagonal texture for the Yukawa couplings will generate a flavor mixing soft-term structure. Therefore, the LFV predictions will impose additional constraints on the free parameters of this kind of models. In this work we concentrate ourselves on two examples of string-inspired models with non-universal soft terms. The relevance of the non-universality of the trilinear terms (A-terms) has recently been considered in refs. [19, 20, 21]. It was shown that the flavor structure of the A-terms is crucial for enhancing the SUSY contributions to CP violation effects, and for generating the experimentally observed ε and ε'/ε. These models also predict a CP asymmetry in $B \to X_s\gamma$ decay much larger than the one predicted by SM and the one obtained for a wide region of the parameter space in minimal supergravity scenarios [22]. It has also been noted that the analysis of $b \to s\gamma$ does not severely constrain the models under consideration [23]. However, the non-universality of the A-terms in this class of models is always associated with non-universal scalar masses. In this case a simple non-diagonal Yukawa texture predicting lepton masses will induce two sources of LFV: one due to the flavor structure of the A_l-terms which prohibits the simultaneous diagonalization of the lepton Yukawa matrix Y_l and the trilinear couplings $(Y_l^A)_{ij} \equiv (A_l)_{ij}(Y_l)_{ij}$; the other source is due to the non degeneracy of the scalar masses of the sleptons. Therefore, in the basis where m_l is diagonal the slepton mass matrix acquires non-diagonal contributions. We find that in general the second source dominates over the first in the case of the LFV predictions.

We analyze the dependence of our results on the lepton Yukawa texture. We study two different *Ansätze* for Y_l, leading to the correct prediction for the charged lepton masses. The form of Y_l can be further specified when a predictive mechanism for the neutrino sector is included in the models. However we do not address this issue here. As we will see, the structure of Y_l is decisive in determining to what extent the soft terms generated by the models can deviate from the universal case.

The paper is organized as follows. In section 2 we present the structure of the soft terms in the two string models we analyze. In section 3 we present the two examples of lepton Yukawa couplings and the slepton mass matrices. In section 4 we present the theoretical framework for calculating the LFV in this class of models. Our predictions for charged leptonic rare processes are given in section 5 for weakly coupled heterotic string (WCHS), and those for the type-I string model are given in section 6. Finally we give our conclusions in section 7.

2. Flavor structure of soft SUSY breaking parameters

In this section we briefly discuss the possible mechanisms which may give rise to the non-universal soft SUSY breaking terms. As already mentioned, motivated by the minimal supergravity model, it is common to assume, that the soft SUSY breaking terms are universal at the GUT scale. However, it is possible to obtain effective potentials in which this universality is absent, as it is the case of considering the kinetic terms for the chiral superfields to be non-minimal. As recently stressed in refs. [14, 15], the soft SUSY breaking parameters may be non-universal in the effective theories derived from superstring theories.

The problem of SUSY breaking is not completely understood yet, even in superstring theories. However, generic superstring models include a dilaton field S and moduli fields T_i, these are gauge singlet fields with their coupling to the gauge non-singlet matter being suppressed by powers of Planck mass. Therefore, they can naturally constitute a 'hidden sector'. Recently [14, 15], the soft SUSY breaking terms have been derived under the assumption that SUSY is broken only by the vacuum expectation values (VEV's) of F-terms corresponding to these S and T_i fields.

This general framework has been used to study the implications on the effective supergravity theories which emerge in the low energy limit of the weakly coupled heterotic strings (WCHS) [14] and type I string models [15]. These two examples show that the soft SUSY breaking terms are, in general, non-universal. The details for deriving these soft terms are given in ref. [14, 15] and some aspect of their phenomenological implications can be found in ref. [19]. Here we briefly present the soft terms which are essential for our work.

In the WCHS case, it is assumed that the superpotential of the dilaton (S) and moduli (T) fields is generated by some non-perturbative mechanism, and that the F-terms of S and T contribute to the SUSY breaking. Hence one can parametrize the F-terms as [14]

$$F^S = \sqrt{3} m_{3/2} (S + S^*) \sin\theta, \qquad F^T = m_{3/2} (T + T^*) \cos\theta \,. \tag{2.1}$$

Here $m_{3/2}$ is the gravitino mass and $\tan\theta$ corresponds to the ratio between the F-terms of S and T. In this framework, the soft scalar masses m_i and the gaugino masses M_a are given by [14]

$$m_i^2 = m_{3/2}^2(1 + n_i \cos^2\theta)\,, \tag{2.2}$$

$$M_a = \sqrt{3} m_{3/2} \sin\theta\,, \tag{2.3}$$

where n_i is the modular weight of the corresponding field. The A-terms are written as

$$A_{ijk} = -\sqrt{3} m_{3/2} \sin\theta - m_{3/2} \cos\theta (3 + n_i + n_j + n_k)\,, \tag{2.4}$$

where $n_{i,j,k}$ are the modular weights of the fields that are coupled by this A-term. It is worth mentioning that in the derivation of eq. (2.4) we have assumed that the Yukawa couplings are T-independent (this is indeed the case in many string models [14]), hence we neglected the terms in A which arise from variying the logarithm of the Yukawa couplings respect to the moduli fields. If we assume $n_i = -1$ for the third family and $n_i = -2$ for the first and second families (in addition we take $n_{H_1} = -1$ and $n_{H_2} = -2$), we find the following texture for the A_l-parameter matrix at the string scale

$$A_l = \begin{pmatrix} x & x & y \\ x & x & y \\ y & y & z \end{pmatrix}\,, \tag{2.5}$$

where

$$x = m_{3/2}(-\sqrt{3}\sin\theta + 2\cos\theta)\,, \tag{2.6}$$

$$y = m_{3/2}(-\sqrt{3}\sin\theta + \cos\theta)\,, \tag{2.7}$$

$$z = -\sqrt{3} m_{3/2} \sin\theta\,. \tag{2.8}$$

Our choice of modular weights is motivated from the fact that assigning different values of the modular weights for the first and second families would make their scalar masses non degenerate, this would imply in general values for $BR(\mu \to e\gamma)$ in conflict with the experimental limit (1.1). Conversely, assigning a common modular weight for all the families would lead to degenerate scalar masses as well as universal A-terms and hence lepton flavor would be preserved.

The deviation from the universality of the soft-terms in this model can be parameterized by the angle θ. The value $\theta = \pi/2$ corresponds to the universal limit for the soft terms. In order to avoid negative squared values in the scalar masses we restrict ourselves to the case with $\cos^2\theta < 1/2$. Such restriction on θ makes the deviation from the universality in the whole soft SUSY breaking terms very limited. However, as shown in [19], this small deviation from the universality of the soft terms is enough to generate sizeable SUSY CP violations in the $K^0 - \overline{K}^0$ system.

In type-I string model case, non-universality in the scalar masses, A-terms and gaugino masses may be naturally obtained [15]. Type-I models can contain 9-branes, 5_i-branes, 7_i-branes and 3 branes where the index $i = 1, 2, 3$ denote the complex compact coordinate which is included in the 5-brane world volum or which is orthogonal to the 7-brane world volume. However, to preserve $N = 1$ supersymmetry in $D = 4$ not all of these branes can be present simultaneously and we can have either $D9$-branes with $D5_i$-branes or $D3$-branes with $D7_i$-branes. From the phenomenological point of view there is no difference between these two scenarios. Here we consider the model used in ref. [20], where the gauge group $\mathrm{SU}(3)_C \times \mathrm{U}(1)_Y$ is associated with the 9-brane while $\mathrm{SU}(2)_L$ is associated with the 5_1-brane.

The SUSY breaking is analyzed, as in WCHS model, in terms of the VEV's of the dilaton and moduli fields [15]

$$F^S = \sqrt{3} m_{3/2} (S + S^*) \sin\theta\,, \qquad F^{T_i} = m_{3/2} (T_i + T_i^*) \Theta_i \cos\theta\,, \tag{2.9}$$

where $i = 1, 2, 3$, the angle θ and the parameters Θ_i just parametrize the direction of the goldstino in the S and T_i fields space. The parameters Θ_i verify the relation,

$$\sum_i |\Theta_i|^2 = 1\,. \tag{2.10}$$

Within this framework, the gaugino masses are [15]:

$$M_1 = M_3 = \sqrt{3} m_{3/2} \sin\theta\,, \tag{2.11}$$

$$M_2 = \sqrt{3} m_{3/2} \Theta_1 \cos\theta\,. \tag{2.12}$$

In this model the fermion doublets and the Higgs fields are assigned to the open string which spans between the 5_1-brane and 9-brane, while the fermion singlets correspond to open strings which start and end on the 9 brane. Such open string states include three sectors which correspond to the three complex compact dimensions. If we assign the fermion singlets to different sectors we obtain non-universal A-terms. It turns out that in this model the trilinear couplings are given [15, 20] by:

$$A_u = A_d = A_l = \begin{pmatrix} x & y & z \\ x & y & z \\ x & y & z \end{pmatrix}, \tag{2.13}$$

where

$$x = -\sqrt{3} m_{3/2} \left(\sin\theta + (\Theta_1 - \Theta_3)\cos\theta\right), \tag{2.14}$$

$$y = -\sqrt{3} m_{3/2} \left(\sin\theta + (\Theta_1 - \Theta_2)\cos\theta\right), \tag{2.15}$$

$$z = -\sqrt{3} m_{3/2} \sin\theta\,. \tag{2.16}$$

The soft scalar masses for sfermion-doublets (m_L^2), and the sfermion-singlets ($m_{R_i}^2$) are given by

$$m_L^2 = m_{3/2}^2 \left(1 - \frac{3}{2}(1 - \Theta_1^2)\cos^2\theta\right), \tag{2.17}$$

$$m_{R_i}^2 = m_{3/2}^2 \left(1 - 3\Theta_i^2 \cos^2\theta\right), \tag{2.18}$$

where $i = 1, 2, 3$ refers to the three families [20]. The soft masses for the Higgs fields are similar to the one of the sfermion-doublets (2.17). For $\Theta_i = 1/\sqrt{3}$ the A-terms and the scalar masses are universal while the gaugino masses can be non-universal. The universal gaugino masses are obtained for $\theta = \pi/6$.

3. Slepton mass matrices and Yukawa textures

In order to completely specify the models described in the previous section, we have to fix the Yukawa textures and hence determine the flavor structure of the slepton mass matrices. In string theories, in principle, Yukawa couplings are calculable. Futhermore, some realistic Yukawa textures can be derived from specific string models [17, 18]. However, in our analysis we will follow a phenomenological approach by using some typical quark and lepton Yukawa textures which we assume to be written in the same basis as the A-terms in eqs. (2.5) and (2.13).

In refs. [19], some phenomenological consequences of the flavor dependence of the squark soft terms were studied. These works assume some typical quark Yukawa textures with satisfactory predictions for quark masses and mixings. Similarly, a general Yukawa texture for the leptonic sector will translate the flavor dependence of the soft terms at the GUT scale into flavor mixing lepton-slepton vertices.

We should emphasize that the only experimental constraint on the lepton Yukawa couplings in the context of the MSSM with flavor blind soft terms (i.e. as derived from minimal supergravity) is the correct prediction for the lepton masses. However, with soft terms as described in the previous section, the results stated in the present work will depend strongly on the structure of the Yukawa texture assumed for the lepton sector.

To illustrate the dependence of the next sections results on the lepton Yukawa couplings, we consider two examples of symmetric textures at the GUT scale:

- Texture I,

$$Y_l = y^\tau \begin{pmatrix} 0 & 5.07 \times 10^{-3} & 0 \\ 5.07 \times 10^{-3} & 8.37 \times 10^{-2} & 0.4 \\ 0 & 0.4 & 1 \end{pmatrix} \tag{3.1}$$

- Texture II,

$$Y_l = y^\tau \begin{pmatrix} 3.3 \times 10^{-4} & 1.64 \times 10^{-5} & 0 \\ 1.64 \times 10^{-5} & 8.55 \times 10^{-2} & 0.4 \\ 0 & 0.4 & 1 \end{pmatrix}. \tag{3.2}$$

Both of them lead to the correct prediction for the experimental values of the lepton masses.

Texture I is a symmetric texture used in some of the solutions obtained in [24]. Our motivation is that this texture can be considered to be the limiting case of textures arising from U(1) family symmetries as described in refs. [25] and studied in refs. [7] in the context of LFV induced by R-H neutrinos. As we will see, when this texture is considered, the BR($\mu \to e\gamma$) imposes a severe constraint on the parameter space of the models, allowing a very small deviation from the universality on the soft terms. Typically a prediction for the decay $\tau \to \mu\gamma$ of the order of the experimental limit (1.1) will imply a severe violation of the experimental bound for $\mu \to e\gamma$.

Texture II provides a good prediction for the lepton masses and induces branching ratios $\mu \to e\gamma$ and $\tau \to \mu\gamma$ of the order of the current experimental bounds. This texture was chosen as an illustration of how the current bounds (1.1) can provide some information about the lepton Yukawa couplings on the context of the models considered.

We should stress that texture I can fit in a more complete model of Yukawa matrices, as the ones aimed to explain fermion masses and quark mixings with a minimal amount of input parameters. On the other hand, we selected texture II based on the predictions for the processes under consideration. It is beyond the purpose of our work to include this texture in the context of a general model for the Yukawa couplings. However, GUT theories such as $\mathrm{SU}(3)_c \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ can provide an example of lepton Yukawa couplings not related to those of the quarks (see for example ref. [26]). We must also observe that unlike texture I, the structure of texture II cannot be similar to the ones obtained in the models with U(1)-family symmeties as described in refs. [25, 26], since it is not possible to arrange the U(1)-charges in order to have the element $(1,1)$ of the texture larger than the $(1,2)$ and $(1,3)$ elements.

The texture assumed for the quark Yukawa couplings has a marginal effect on the computation of the branching ratios studied in the present work. Nevertheless, we need the full matricial structure of all Yukawa couplings in order to be consistent with our analysis of the renormalization-group equations (RGE's). Therefore, we use the *Ansätze* given in solution 2 of ref. [24], with inputs at the GUT scale leading to the correct experimental predictions for the quark sector.

The lepton Yukawa couplings can be diagonalized by the unitary matrices U_L and U_R as follows,

$$m_l = \frac{v\cos\beta}{\sqrt{2}} U_R\ (Y^l)^T\ U_L^\dagger\,. \tag{3.3}$$

When the superfields are written in this basis, the expressions for the charged slepton mass matrices at low energy take the form:

$$M_{\tilde{l}}^2 = \begin{pmatrix} \left(M_{\tilde{l}}^2\right)_{LL} & \left(M_{\tilde{l}}^2\right)_{LR} \\ \left(M_{\tilde{l}}^2\right)_{RL} & \left(M_{\tilde{l}}^2\right)_{RR} \end{pmatrix}, \tag{3.4}$$

where,

$$\begin{aligned} \left(M_{\tilde{l}}^2\right)_{LL} &= U_L m_L^2 U_L^\dagger + m_l^2 - \frac{m_Z^2}{2}(1 - 2\sin^2\theta_W)\cos 2\beta\,, \\ \left(M_{\tilde{l}}^2\right)_{RR} &= U_R (m_R^2)^T U_R^\dagger + m_l^2 + m_Z^2 \sin^2\theta_W \cos 2\beta\,, \\ \left(M_{\tilde{l}}^2\right)_{LR} &= \left(M_{\tilde{l}}^2\right)_{RL}^\dagger = -\mu\ m_l \tan\beta + \frac{v\cos\beta}{\sqrt{2}} U_L Y_l^{A*} U_R^\dagger\,, \end{aligned} \tag{3.5}$$

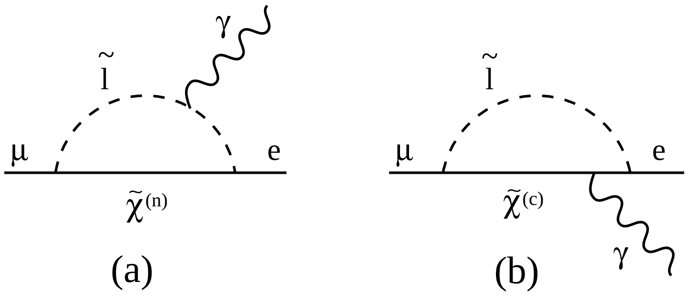

Figure 1: The generic Feynman diagrams for $\mu \to e\gamma$ decay. $\tilde{l}$ stands for charged slepton (a) or sneutrino (b), while $\tilde{\chi}^{(n)}$ and $\tilde{\chi}^{(c)}$ represent neutralinos and charginos respectively.

where m_L^2 and m_R^2 are the soft breaking (3×3) mass matrices for the charged slepton doublet and singlet respectively.

The sneutrino mass matrix is simply given by the (3×3) mass matrix:

$$M_{\tilde{\nu}}^2 = U_L m_L^2 U_L^\dagger + \frac{m_Z^2}{2} \cos 2\beta \,. \tag{3.6}$$

The relevant lepton-flavor changing mass matrix elements on the slepton mass matrices above are given by:

$$\begin{aligned}(\delta^l_{LL})_{ij} &= \Big[\ U_L\ m_L^2\ U_L^\dagger\ \Big]_{ij} \\ (\delta^l_{LR})_{ij} &= \Big[\ U_L\ Y_l^{A*}\ U_R^\dagger\ \Big]_{ij} \\ (\delta^l_{RR})_{ij} &= \Big[\ U_R\ (m_R^2)^T\ U_R^\dagger\ \Big]_{ij}\,,\end{aligned} \tag{3.7}$$

where i, j are flavor indices $(i \neq j)$. We found that, in general, δ^l_{LL} and δ^l_{RR} are much more enhanced by the non degeneracy of the scalar soft masses that what δ^l_{LR} is due to the non-universality of the A-terms.

4. LFV in SUSY models

Figure 1 shows the one-loop diagrams that are relevant to the $\mu \to e\gamma$ process. The corresponding $\tau \to \mu\gamma$ can be represented by analogous graphs. The amplitude for the decay can be written as a magnetic transition:

$$\mathcal{T}(l_j \to l_i\gamma) = e\ \epsilon^\lambda \bar{u}_i(p-q)\{m_j i\sigma_{\lambda\beta}q^\beta \left(A^L P_L + A^R P_R\right)\}u_j(p)\,, \tag{4.1}$$

where q is the photon momentum. A^L and A^R receive contributions from both neutralino-charged slepton (n) and chargino-sneutrino (c) exchange

$$A^{L,R} = A_n^{L,R} + A_c^{L,R}\,. \tag{4.2}$$

In the limit of vanishing mass for the outgoing leptons, eq. 4.1 can be written as

$$\mathcal{T}(l_j \to l_i\gamma) = e\ \bar{u}_i(p-q)\{2p\cdot\epsilon\ m_j \left(A^L P_L + A^R P_R\right)\}u_j(p)\,. \tag{4.3}$$

Thus the decay rate is given by:

$$\Gamma(l_j \to l_i\gamma) = \frac{m_{l_j}^3}{16\pi}(|A_R|^2 + |A_L|^2)\,. \tag{4.4}$$

The formulae used to calculate these amplitudes can be found in ref. [6]. The evaluation of the branching ratios for the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays involve the masses of a good part of the supersymmetric particles. Therefore, it is important to know precisely all masses and other low energy parameters for any given set of inputs at the GUT scale (M_{GUT}). In the present work, this is obtained by numerical integration of the RGE's of the MSSM. The complete RGE's at two loops can be found in ref. [27].

In addition to the soft-breaking terms dictated by the string-models under consideration, our effective theory below M_{GUT} depends on the parameters:

$$\alpha_G,\ M_{GUT},\ \tan\beta,\ Y_u,\ Y_d,\ Y_l\,.$$

The quantities $\alpha_G = g_G^2/4\pi$ (g_G being the GUT gauge coupling constant) and M_{GUT} are evaluated consistently with the experimental values of α_{em}, α_s, and $\sin^2\theta_W$ at m_Z. We integrate numerically the RGE's for the MSSM at two loops in the gauge and Yukawa couplings and at one loop in the soft terms, from M_{GUT} down to a common supersymmetric threshold $M_S \approx 200\,\mathrm{GeV}$. From this energy to m_Z, the RGEs of the SM are used. The value of the B and μ parameter (up to its sign) can be expressed in terms of the other input parameters by means of the electroweak symmetry breaking conditions. We fix the elements of the Yukawa matrices at the GUT scale, consistently with the experimental values for the fermion masses and the absolute values of CKM matrix elements. Despite the fact that we use the full matrix form for all parameters, our results do not differ significantly from the common approach of considering diagonal Yukawa matrices and neglecting the two lightest generations. We must observe, however, that in the models here discussed, the trilinear terms are not diagonal in the basis where the Yukawa couplings are, and therefore one can not avoid a matricial treatment for them. Finally we keep a fixed value for $\tan\beta = 10$ and set the sign of the μ parameter to be positive. As we can see in previous studies [6, 7], the branching ratios under consideration increase with $\tan\beta$.

5. $BR(l_j \to l_i\gamma)$ in the WCHS model

As stated in section 2, the non-universality of the soft terms in the WCHS arises from the different modular weight assigned to each family and the parameter θ. Hence the non-universalities in eqs. (2.2) and (2.5) will translate into non-trivial values for the flavor-changing matrix elements given in eq. (3.7). Therefore due to presence of flavor-mixing elements in the charged and neutral slepton mass matrices, the two diagrams of figure 1 contribute to the four partial amplitudes of eq. (4.2).

JHEP07(2001)001

The choice of input values for the SUSY parameters described in section 4 is completed once we give the model-dependent values of the soft masses and trilinear terms. In order to do that we need to fix the values of $m_{3/2}$ and the angle θ in eqs. (2.2)–(2.5). The splitting of the soft masses increases from $\sin\theta = 1$ (which corresponds to the universal case) to the limiting case for $\sin\theta = 1/\sqrt{2}$ (below which some square masses become negative). Therefore we consider as representative for the WCHS model to present the variation of the branching ratios with $\sin\theta$ for fixed values of $m_{3/2}$ as shown in figure 2. For the value $m_{3/2} = 200\,\mathrm{GeV}$, the mass of lightest neutralino varies from 100 to 147 GeV, that of the chargino from 190 to 277 GeV, while the lightest of the staus has masses of 107 to 233 GeV as $\sin\theta$ ranges from $1/\sqrt{2}$ to 1. Similarly for $m_{3/2} = 400\,\mathrm{GeV}$ we found $m_{\tilde{\chi}^0} = 210 - 295\,\mathrm{GeV}$, $m_{\tilde{\chi}^+} = 400 - 570\,\mathrm{GeV}$, $m_{\tilde{\tau}_2} = 212 - 470\,\mathrm{GeV}$ for the same range of $\sin\theta$.

Figure 2 shows the results of the branching ratios under consideration. We can see how texture I (graphic on top) tolerates small deviations from universality of the soft terms. The experimental bound on $BR(\mu \to e\gamma)$ is satisfied only for $\sin\theta > .96$ ($m_{3/2} = 200\,\mathrm{GeV}$) and for $\sin\theta > .91$ ($m_{3/2} = 400\,\mathrm{GeV}$) while for the same range on $\sin\theta$ the corresponding prediction for $BR(\tau \to \mu\gamma)$ is well below the experimental bound. The values of the branching ratios decrease as we increase $m_{3/2}$ since this translates into an increase of the masses of the supersymmetric particles. In order to simplify the presentation of our results we fix $\tan\beta = 10$. However, enlarging the value of $\tan\beta$ increases the prediction for the branching ratios as shown for example in refs. [6, 7].

The results obtained using texture II (figure 2, bottom graphic) allow us to start the graph at the lowest value of $\sin\theta = 1/\sqrt{2}$. As it can be seen, the experimental bounds are more restrictive for the $\tau \to \mu\gamma$ than for $\mu \to e\gamma$ process.

We find values of the same order for the partial amplitudes A_n^L, A_n^R and A_c^R of eq. (4.2) that contribute to the decay in eq. (4.4). A_c^L arises due to Yukawa interactions and is roughly three orders of magnitude smaller than the other amplitudes. The relative sign of the amplitudes depends on the texture and values of the input parameters. We observe accidental cancellations of partial amplitudes for values of $\sin\theta$ close to 1. Still, the effect is not very important, since the amplitudes are very small for these values of $\sin\theta$ and therefore hard to observe in figure 2. The flavor mixing elements introduced by δ^l_{LL} and δ^l_{RR} (3.7) in the scalar matrices, have a larger impact on the amplitudes than those due to the trilinear terms δ^l_{LR}.

6. $BR(l_j \to l_i\gamma)$ in the type-I string model

The structure of the soft-terms in the type-I string model is more complicated than in the previous model. They depend, in addition to $m_{3/2}$ and θ, on the values of the parameters Θ_1, Θ_2 and Θ_3. However, the flavor structure of the slepton matrices is simpler, since the soft masses for the left-handed sleptons of eq. (2.17) are universal

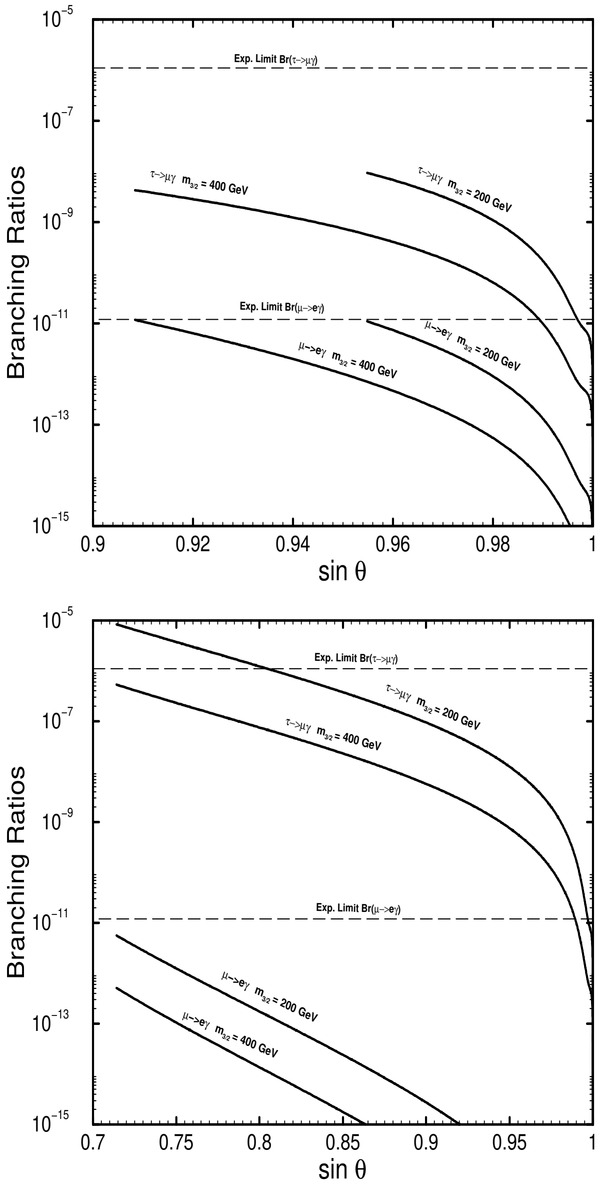

Figure 2: Branching ratios vs. $\sin\theta$ for the WCHS model with texture I for Y_l (top) and texture II (bottom) and $\tan\beta = 10$. The values for $m_{3/2}$ are kept constant as shown on the curves.

at the GUT scale and the sneutrino mass matrix of eq. (3.6) remains diagonal under a rotation that diagonalizes Y_l. Therefore diagram (b) in figure 1 does not contribute to the LFV processes calculated here.

The restrictions imposed by the experimental limits on the searches for charginos and sleptons will constrain the free parameters of the model. A bound of $m_{\tilde{\chi}^+} = 95\,\mathrm{GeV}$ is found to be the most severe on the initial conditions of eqs. (2.11) and (2.12). The restrictions imposed by the bound of $90\,\mathrm{GeV}$ on the mass of the lightest charged slepton will impose constraints on eqs. (2.17) and (2.18). These constraints depend on the values of the Θ_i's as we will discuss later.

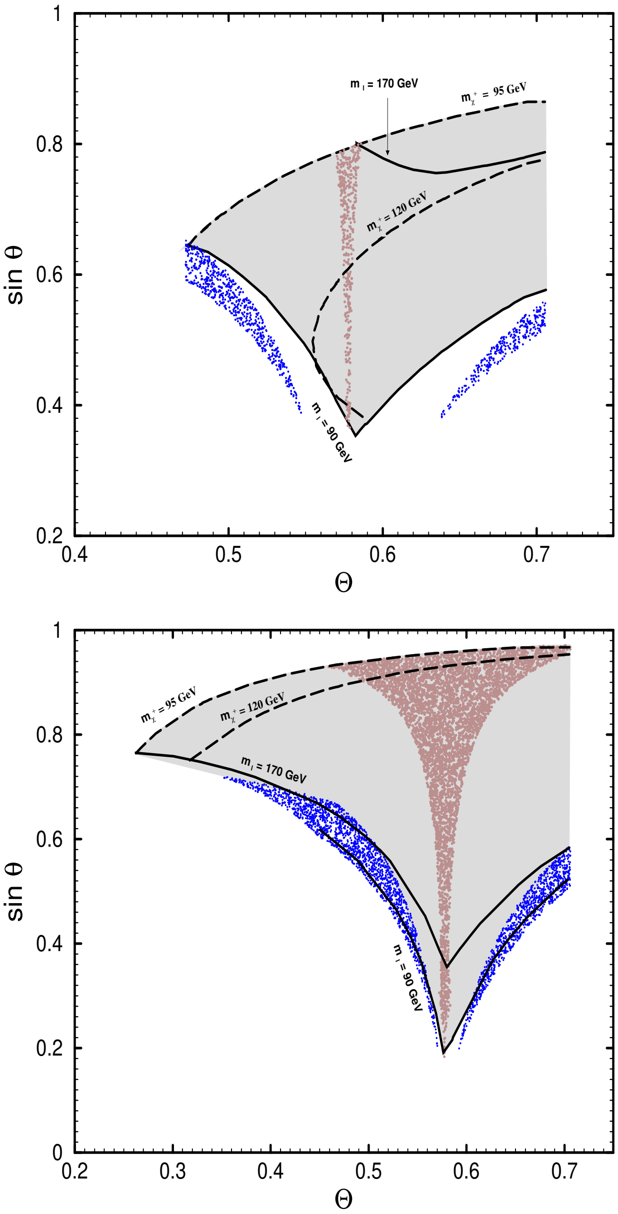

Figure 3: Areas with $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ (dotted areas inside the gray contour) in the plane $\sin\theta - \Theta$ for constant values of $m_{3/2} = 200\,\mathrm{GeV}$ (top) and $m_{3/2} = 400\,\mathrm{GeV}$ (bottom) and $\tan\beta = 10$. The model used corresponds to type-I string, with texture I for Y_l. Values of the masses of SUSY particles which bound the parameter space of the model are as shown in the graphs. The dark dotted areas correspond to space of parameters such that the LSP is a slepton.

The predictions we obtain with this model for $BR(l_j \to l_i\gamma)$ allow us to simplify our presentation by setting $\Theta \equiv \Theta_1 = \Theta_2$. In the case of texture I, this is justified by the fact that the experimental bound on $BR(\mu \to e\gamma)$ tolerates a small deviation of the Θ_i 's from the common value of $1/\sqrt{3}$, for which the soft masses become universal (see figure 3). For the case of texture II, these predictions are more tolerant to a

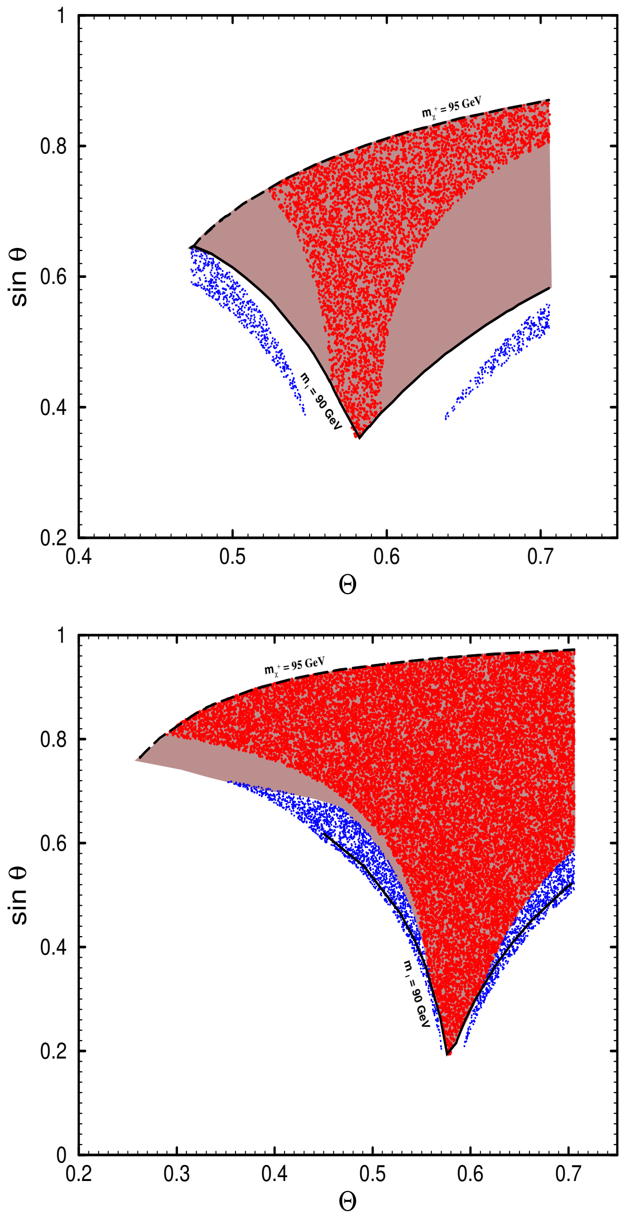

Figure 4: Areas with $BR(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$ (dotted areas inside the gray contour) in the plane $\sin\theta - \Theta$ for constant values of $m_{3/2} = 200\,\mathrm{GeV}$ (top) and $m_{3/2} = 400\,\mathrm{GeV}$ (bottom) and $\tan\beta = 10$. The model used corresponds to type-I string, with texture II for Y_l. Values of the masses of SUSY particles which bound the parameter space of the model are as shown in the graphs. The dark dotted areas correspond to space of parameters such that the LSP is a slepton.

variation of the Θ_i 's. However, when this texture is considered, the experimental limit on $BR(\tau \to \mu\gamma)$ is more restrictive (see figures 4, 5 and 6), since this bound is particularly sensitive to the value Θ_3. Therefore, we find that by setting also $\Theta_1 = \Theta_2$ in the analysis of our results with texture II we can achieve a clearer presentation without any loss of generality.

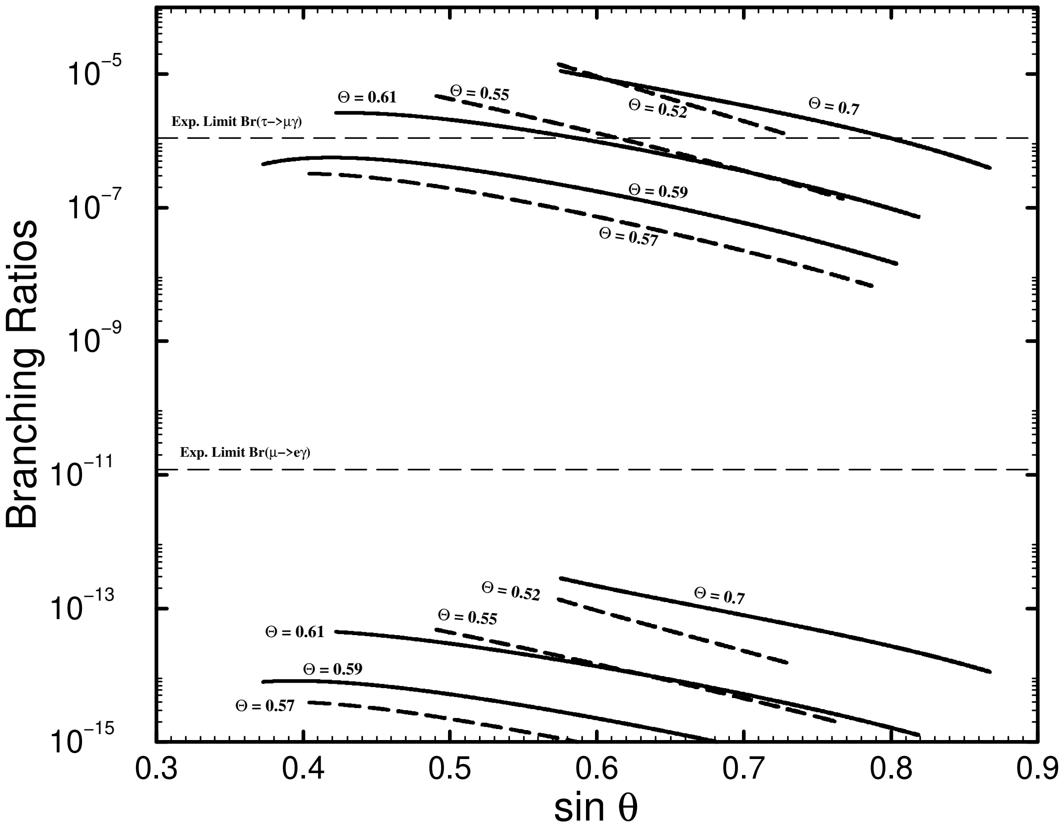

Figure 5: Branching ratios vs. $\sin\theta$ for the type-I string model with texture II for Y_l, $m_{3/2} = 200\,\mathrm{GeV}$ and $\tan\beta = 10$. The values for Θ are kept constant as shown on the curves, solid (dashed) lines correspond to values for $\Theta > 1/\sqrt{3}(\Theta < 1/\sqrt{3})$.

Figure 3 shows the constraint imposed by the current bound on the $BR(\mu \to e\gamma)$ on the plane $(\sin\theta - \Theta)$ for constant values of $m_{3/2} = 200\,\mathrm{GeV}$ (top) and $m_{3/2} = 400\,\mathrm{GeV}$ (bottom) when texture I is assumed. Figure 4 displays the equivalent for texture II. The light shaded areas correspond to the space of parameters allowed by the bounds on the masses of the SUSY particles. The region below the upper dashed line corresponds to values of $m_{\tilde{\chi}^+} > 95\,\mathrm{GeV}$, while the sector above the lower solid line corresponds to values of the lightest charged scalar $m_{\tilde{l}} > 90\,\mathrm{GeV}$.

The shape of the curve $m_{\tilde{l}} = 90\,\mathrm{GeV}$ in figures 3 and 4 is determined by the initial conditions given by eqs. (2.17) and (2.18). The lowest values for these masses corresponds to $m_{R_1} = m_{R_2}$ when $1/\sqrt{3} < \Theta < 1/\sqrt{2}$, while for $\Theta < 1/\sqrt{3}$, m_{R_3} is the lowest value. Therefore the largest component of the lowest eigenvalue of the charged slepton mass is the $\tilde{e}_R$ or the $\tilde{\tau}_R$ depending on the ranges of Θ above. Similar considerations explain the different shape of the curves for $m_{\tilde{l}} = 170\,\mathrm{GeV}$ (with $m_{3/2} = 200\,\mathrm{GeV}$ and $m_{3/2} = 400\,\mathrm{GeV}$).

The darkest dotted areas in figures 3 and 4 represent the sector of parameters for which the lightest supersymmetric particle (LSP) is a charged slepton. For $m_{3/2} =$

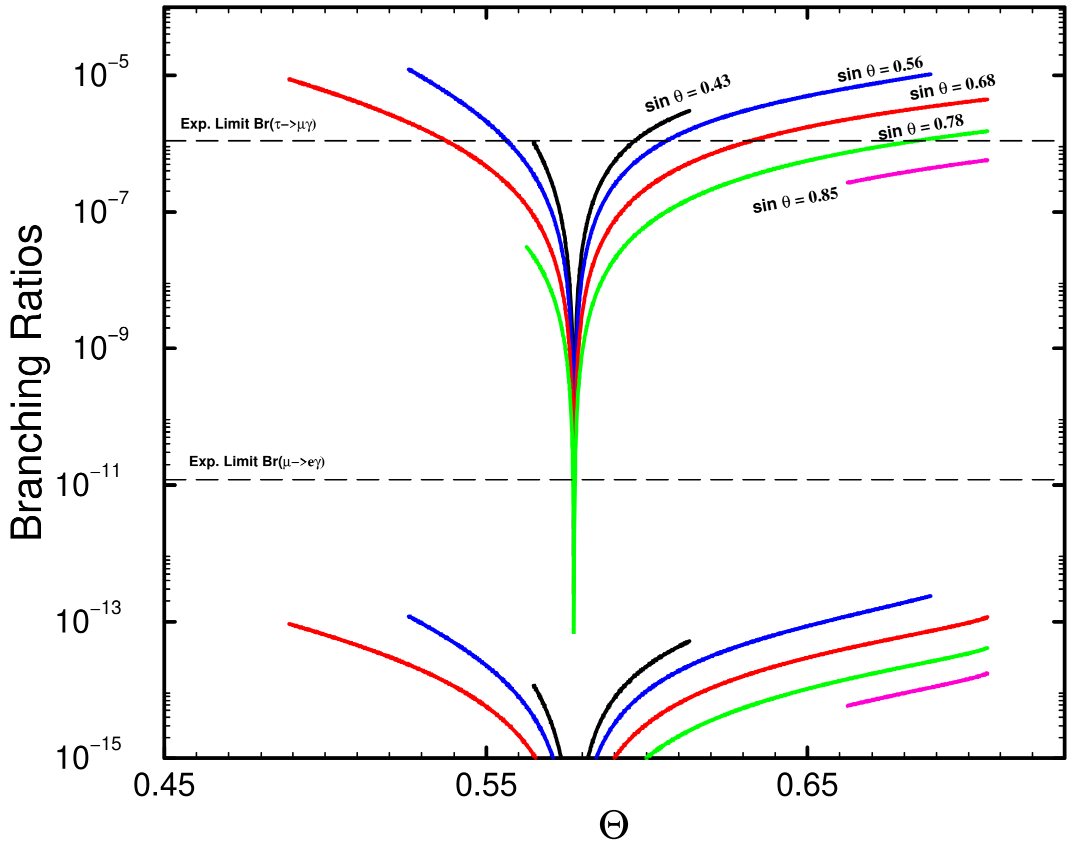

Figure 6: Branching ratios vs. Θ for the type-I string model with texture II for Y_l, $m_{3/2} = 200\,\text{GeV}$ and $\tan\beta = 10$. The values for $\sin\theta$ are kept constant as shown on the curves.

200 GeV these areas are below the bound of $m_{\tilde{l}} = 90\,\text{GeV}$. However for $m_{3/2} = 400\,\text{GeV}$, the cosmological requirement on the LSP to be a neutral particle (lightest neutralino in our case) imposes a further restriction on the space of parameters of the model.

Similarly to the results found for the WCHS model, the assumption of texture I for Y_l allows a small deviation from the universality of the scalar masses once we impose the experimental bound on $BR(\mu \to e\gamma)$ (light dotted sector inside of the grey area in figure 3). However we found that the corresponding limit on $\tau \to \mu\gamma$ does not constraint the space of parameters shown in figure 3. The fact that the branching ratios decrease with $m_{3/2}$ is reflected in a wider light dotted area on the graphic corresponding to $m_{3/2} = 400\,\text{GeV}$ in figure 3.

The flavor mixing elements in the scalar charged scalar mass matrix are introduced by δ^l_{RR} and δ^l_{LR} in eq. (3.7). As stated before, only diagram (a) from figure 1 contributes to the rare lepton decays under consideration. The non-universality of the right sleptons of eq. (2.18) enhances the partial amplitude A^L_n in eq. (4.2) over the A^R_n which is due to the mixing δ^l_{LR} originated from the flavor dependent structure of the A-terms of eq. (2.13). The values we find for A_n^R are of the order of 10% of the values of A_n^L. We stress that a particular choice of the Θ_i's such that $|\Theta_i| = 1/\sqrt{3},\ i = 1,2,3$ with Θ_2 or Θ_3 negative, will maintain the universality of the scalar masses while producing a maximal non-universality in the A-terms (see eq. (2.16)). In this particular case the mixing introduced by δ^l_{LR} can induce branching ratios of the order of the ones here presented.

When texture II is assumed for Y_l, the phenomenological constraints imposed by SUSY particles on the parameter space of the model do not differ significantly from the case of texture I. Lines corresponding to constant masses for sleptons and charginos in figure 4 would be located in the same places as in figure 3. The present limits on $BR(\mu \to e\gamma)$ do not impose any restriction on the parameter space shown in figure 4. However, in the case of texture II, the limits on the decay $\tau \to \mu\gamma$ restrict the space of parameters to the dotted region inside of the gray area on the graphics of figure 4. As one can immediately see, this constraint is significant for the case of $m_{3/2} = 200\,\mathrm{GeV}$ and decreasingly restrictive as $m_{3/2}$ increases to 400 GeV. From these figures one can induce the effect of improving the present experimental limits on the two processes considered.

Figure 5 shows the behavior of the branching ratios with $\sin\theta$ for fixed values of Θ, for the case of texture II and $m_{3/2} = 200\,\mathrm{GeV}$. These ratios decrease as $\sin\theta$ increases since higher values of this parameter are related to larger values of the masses of the supersymmetric particles present on diagram (a) of figure 1. As the values of Θ get closer to $1/\sqrt{3}$ the prediction for the ratios is smaller and will eventually vanish for the limit of universal soft terms corresponding to $\Theta = 1/\sqrt{3}$. We observe that the allowed range of $\sin\theta$ is larger for values of $\Theta > 1/\sqrt{3}$ (solid lines), than for values of $\Theta < 1/\sqrt{3}$, which is explained by the asymmetry of the shaded regions on figures 3 and 4.

Figure 6, in a complementary way to figure 5, shows the behavior of the branching ratios with Θ for fixed values of $\sin\theta$ also for the case of texture II and $m_{3/2} = 200\,\mathrm{GeV}$. As one can see, the universality of the soft terms obtained for $\Theta = 1/\sqrt{3}$ leads to vanishing ratios. As the values from Θ differ from this value the ratios increase. As we mentioned before, higher values of $\sin\theta$ correspond to lower values of the ratios. The ranges on Θ for different values of $\sin\theta$ can be understood from the allowed space of parameters represented on figures 3 and 4.

7. Conclusions

We have studied the predictions for the LFV decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ arising from non universal soft terms as they appear when the MSSM is derived from a general string theory. We studied the dependence of these predictions on a general, non-diagonal, texture of the lepton Yukawa couplings.

The results found show the relevance of the considered processes in constraining the undetermined parameters of the models, and therefore their predictions for the SUSY particles. On the other hand, these processes can provide some information on the charged lepton Yukawa couplings, which can be very important when this models are extended to explain neutrino physics.

We found the non-universality of the soft masses to be more relevant for LFV than those of the A-term are. However, the latter ones are of phenomenological interest for other processes such as CP violation effects. Finally, we would like to emphasize the importance of the improvements on the current experimental limits on LFV processes to understand the nature of the flavor problems on the SUSY extensions of the SM.

Acknowledgments

Our research has been supported by the E.U. T.M.R. Network contract ERBFMRX-CT96-0090 (M.E.G.), PPARC (S. K.) and F.C.T. PRAXIS XXI/BD/9416/96 (D.F.C.). We would like to thank J.C. Romão for his help in the programing stages of our work and for many useful discussions. We also thank D. Bailin and A.M. Teixeira for carefully reading the manuscript.

Note added in proof: After the first version of this work was completed the Muon $(g-2)$ Collaboration had published a new measurement of the anomalous magnetic moment of the muon (H.N. Brown *et al.*, hep-ex/0102017). The result presented differs from the SM prediction by 2.6σ. The MSSM contribution to this process, excludes one of the signs of the μ-parameter ($\mu < 0$ in our convention) for the space of parameters described in this work. The results we presented correspond to $\mu > 0$. However, we should indicate that the a change of sign of the μ-parameter parameter do not change significantly our predictions the branching ratios.

References

[1] Super-Kamiokande collaboration, Y. Fukuda et al., *Measurement of a small atmospheric ν_μ/ν_e ratio*, *Phys. Lett.* **B 433** (1998) 9 [hep-ex/9803006]; *Study of the atmospheric neutrino flux in the multi-GeV energy range*, *Phys. Lett.* **B 436** (1998) 33 [hep-ex/9805006]; *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003].

[2] CHOOZ collaboration, M. Apollonio et al., *Initial results from the Chooz long baseline reactor neutrino oscillation experiment*, *Phys. Lett.* **B 420** (1998) 397 [hep-ex/9711002].

[3] S.T. Petcov, *The processes $\mu \to e, \gamma$, $\mu \to e, e\bar{e}$, neutrino $\to$ neutrino, γ in the Weinberg-Salam model with neutrino mixing*, *Sov. J. Nucl. Phys.* **25** (1977) 340;

T.-P. Cheng and L.-F. Li, *Muon number nonconservation effects in a gauge theory with v a currents and heavy neutral leptons*, *Phys. Rev.* **D 16** (1977) 1425;
B.W. Lee and R.E. Shrock, *Natural suppression of symmetry violation in gauge theories: muon - lepton and electron lepton number nonconservation*, *Phys. Rev.* **D 16** (1977) 1444;
J.D. Bjorken, K. Lane and S. Weinberg, *The decay $\mu \to e^+, \gamma$ in models with neutral heavy leptons*, *Phys. Rev.* **D 16** (1977) 1474.

[4] Particle Data Group collaboration, D.E. Groom et al., *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[5] R. Barbieri and L.J. Hall, *Signals for supersymmetric unification*, *Phys. Lett.* **B 338** (1994) 212 [hep-ph/9408406];
R. Barbieri, L. Hall and A. Strumia, *Violations of lepton flavor and CP in supersymmetric unified theories*, *Nucl. Phys.* **B 445** (1995) 219 [hep-ph/9501334];
S. Dimopoulos and D. Sutter, *The supersymmetric flavor problem*, *Nucl. Phys.* **B 452** (1995) 496 [hep-ph/9504415];
N. Arkani-Hamed, H.-C. Cheng and L.J. Hall, *Flavor mixing signals for realistic supersymmetric unification*, *Phys. Rev.* **D 53** (1996) 413 [hep-ph/9508288];
P. Ciafaloni, A. Romanino and A. Strumia, *Lepton flavor violations in* SO(10) *with large* $\tan\beta$, *Nucl. Phys.* **B 458** (1996) 3 [hep-ph/9507379];
M.E. Gomez and H. Goldberg, *Lepton flavor violation in SUSY*-SO(10) *with predictive yukawa texture*, *Phys. Rev.* **D 53** (1996) 5244 [hep-ph/9510303];
B. de Carlos, J.A. Casas and J.M. Moreno, *Constraints on supersymmetric theories from $\mu \to e, \gamma$*, *Phys. Rev.* **D 53** (1996) 6398 [hep-ph/9507377];
J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Exact event rates of lepton flavor violating processes in supersymmetric* SU(5) *model*, *Phys. Lett.* **B 391** (1997) 341 [hep-ph/9605296];
S.F. King and M. Oliveira, *Lepton flavor violation in string inspired models*, *Phys. Rev.* **D 60** (1999) 035003 [hep-ph/9804283].

[6] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, *Lepton-flavor violation via right-handed neutrino Yukawa couplings in supersymmetric standard model*, *Phys. Rev.* **D 53** (1996) 2442 [hep-ph/9510309].

[7] M.E. Gomez, G.K. Leontaris, S. Lola and J.D. Vergados, U(1)*-textures and lepton flavor violation*, *Phys. Rev.* **D 59** (1999) 116009 [hep-ph/9810291];
J.R. Ellis, M.E. Gomez, G.K. Leontaris, S. Lola and D.V. Nanopoulos, *Charged lepton flavour violation in the light of the Super- Kamiokande data*, *Eur. Phys. J.* **C 14** (2000) 319 [hep-ph/9911459].

[8] G.K. Leontaris and N.D. Tracas, *Lepton flavour violation in unified models with* U(1)*-family symmetries*, *Phys. Lett.* **B 431** (1998) 90 [hep-ph/9803320];
W. Buchmuller, D. Delepine and F. Vissani, *Neutrino mixing and the pattern of supersymmetry breaking*, *Phys. Lett.* **B 459** (1999) 171 [hep-ph/9904219];

W. Buchmuller, D. Delepine and L.T. Handoko, *Neutrino mixing and flavor changing processes*, *Nucl. Phys.* **B 576** (2000) 445 [hep-ph/9912317];
J.L. Feng, Y. Nir and Y. Shadmi, *Neutrino parameters, abelian flavor symmetries and charged lepton flavor violation*, *Phys. Rev.* **D 61** (2000) 113005 [hep-ph/9911370].

[9] M. Frank, *Bilinear R parity violation parameters from $\mu \to e, \gamma$*, *Phys. Rev.* **D 62** (2000) 015006;
A. de Gouvea, S. Lola and K. Tobe, *Lepton flavor violation in supersymmetric models with trilinear R-parity violation*, *Phys. Rev.* **D 63** (2001) 035004 [hep-ph/0008085].

[10] R. Kitano, *Lepton flavor violation in the Randall-Sundrum model with bulk neutrinos*, *Phys. Lett.* **B 481** (2000) 39 [hep-ph/0002279];
F. del Aguila and J. Santiago, *Universality limits on bulk fermions*, *Phys. Lett.* **B 493** (2000) 175 [hep-ph/0008143].

[11] B. Mukhopadhyaya and A. Raychaudhuri, *K(L) $\to$ mu e: a possible probe on slepton mass splitting?*, *Phys. Rev.* **D 42** (1990) 3215;
A. Belyaev et al., *Charged-lepton flavour violation in kaon decays in supersymmetric theories*, hep-ph/0008276.

[12] J. Hisano, M.M. Nojiri, Y. Shimizu and M. Tanaka, *Lepton flavor violation in the left-handed slepton production at future lepton colliders*, *Phys. Rev.* **D 60** (1999) 055008 [hep-ph/9808410];
I. Hinchliffe and F.E. Paige, *Lepton flavor violation at the LHC*, *Phys. Rev.* **D 63** (2001) 115006 [hep-ph/0010086].

[13] J.R. Ellis and D.V. Nanopoulos, *Flavor changing neutral interactions in broken supersymmetric theories*, *Phys. Lett.* **B 110** (1982) 44;
R. Barbieri and R. Gatto, *Conservation laws for neutral currents in spontaneously broken supersymmetric theories*, *Phys. Lett.* **B 110** (1982) 211;
G.K. Leontaris, K. Tamvakis and J.D. Vergados, *Lepton and flavor violation in SUSY models*, *Phys. Lett.* **B 171** (1986) 412;
F. Borzumati and A. Masiero, *Large muon and electron number violations in supergravity theories*, *Phys. Rev. Lett.* **57** (1986) 961;
L.J. Hall, V.A. Kostelecky and S. Raby, *New flavor violations in supergravity models*, *Nucl. Phys.* **B 267** (1986) 415.

[14] A. Brignole, L.E. Ibáñez and C. Munoz, *Towards a theory of soft terms for the supersymmetric standard model*, *Nucl. Phys.* **B 422** (1994) 125 [hep-ph/9308271].

[15] L.E. Ibáñez, C. Munoz and S. Rigolin, *Aspects of type-I string phenomenology*, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397].

[16] M. Brhlik, L.L. Everett, G.L. Kane and J. Lykken, *A resolution to the supersymmetric CP problem with large soft phases via D-branes*, *Phys. Rev. Lett.* **83** (1999) 2124 [hep-ph/9905215]; *Superstring theory and CP violating phases: can they be related ?*, *Phys. Rev.* **D 62** (2000) 035005 [hep-ph/9908326].

[17] J.A. Casas and C. Munoz, *Yukawa couplings in* SU(3)×SU(2)×U(1)*-y orbifold models*, *Phys. Lett.* **B 212** (1988) 343.

[18] T. Kobayashi and Z. zhong Xing, *Quark mass matrices in superstring models*, *Int. J. Mod. Phys.* **A 13** (1998) 2201 [hep-ph/9712432].

[19] S.A. Abel and J.M. Frere, *Could the MSSM have no CP-violation in the CKM matrix?*, *Phys. Rev.* **D 55** (1997) 1623 [hep-ph/9608251];
S. Khalil, T. Kobayashi and A. Masiero, *CP-violation in supersymmetric model with nondegenerate a- terms*, *Phys. Rev.* **D 60** (1999) 075003 [hep-ph/9903544];
S. Khalil and T. Kobayashi, *Supersymmetric CP-violation ε'/ε due to asymmetric a-matrix*, *Phys. Lett.* **B 460** (1999) 341 [hep-ph/9906374].

[20] S. Khalil, T. Kobayashi and O. Vives, *Edm-free supersymmetric CP-violation with non-universal soft terms*, *Nucl. Phys.* **B 580** (2000) 275 [hep-ph/0003086].

[21] M. Brhlik, L.L. Everett, G.L. Kane and J. Lykken, *A resolution to the supersymmetric CP problem with large soft phases via D-branes*, *Phys. Rev. Lett.* **83** (1999) 2124 [hep-ph/9905215].

[22] D. Bailin and S. Khalil, *Flavor-dependent SUSY phases and CP asymmetry in $B \to X_s\gamma$ decays*, *Phys. Rev. Lett.* **86** (2001) 4227 [hep-ph/0010058];
S. Abel, D. Bailin, S. Khalil and O. Lebedev, *Flavour-dependent CP-violation and natural suppression of the electric dipole moments*, *Phys. Lett.* **B 504** (2001) 241 [hep-ph/0012145].

[23] E. Gabrielli, S. Khalil and E. Torrente-Lujan, *The $b \to s\gamma$ decay in SUSY models with non-universal a-terms*, *Nucl. Phys.* **B 594** (2001) 3 [hep-ph/0005303].

[24] P. Ramond, R.G. Roberts, and G.G. Ross, *Stitching the Yukawa quilt*, *Nucl. Phys.* **B 406** (1993) 19 [hep-ph/9303320].

[25] L. Ibáñez and G.G. Ross, *Fermion masses and mixing angles from gauge symmetries*, *Phys. Lett.* **B 332** (1994) 100 [hep-ph/9403338];
G.K. Leontaris, S. Lola and G.G. Ross, *Heavy neutrino threshold effects in low-energy phenomenology*, *Nucl. Phys.* **B 454** (1995) 25 [hep-ph/9505402].

[26] S. Lola and G.G. Ross, *Neutrino masses from* U(1) *symmetries and the Super-Kamiokande data*, *Nucl. Phys.* **B 553** (1999) 81 [hep-ph/9902283].

[27] S.P. Martin and M.T. Vaughn, *Two loop renormalization group equations for soft supersymmetry breaking couplings*, *Phys. Rev.* **D 50** (1994) 2282 [hep-ph/9311340].

JHEP07(2001)001